Grade 5 3-digit ÷ 1-digit division

5.N.6	
Demonstrate an understanding of division (3-digit numerals by 1-digit numerals) with and without concrete materials, and interpret remainders to solve problems	1. Model the division process as equal sharing using base-10 blocks, and record it symbolically. 2. Explain that the interpretation of a remainder depends on the context: 2.1. ignore the remainder (e.g., making teams of 4 from 22 people) 2.2. round up the quotient (e.g., the number of five passenger cars required to transport 13 people) 2.3. express remainders as fractions (e.g., five apples shared by two people) 2.4. express remainders as decimals (e.g., measurement and money) 3. Solve a division problem in context using personal strategies, and record the process.

Clarification of the outcome:

- The outcome is dividing a 3-digit whole number by a 1-digit whole number and solving problems involving division. Remainders arising in problems are to be treated in realistic ways.
- Efficiency of algorithm is encouraged in the DEVELOP lesson. Clunky procedures are soon left behind by students in favor of the calculator. Can you blame them for doing so?
- The Manitoba curriculum guide suggests using base-ten materials in achievement indicator #1. The reality is that these materials are clunky to work with for division. Inevitably (unless questions are carefully crafted so that the number of ten-pieces is exactly divisible by the divisor), a ten-piece will need to be traded (broken up) for ten one-pieces. It is less distracting and just as effective to simply use counters instead.

Required close-to-at-hand prior knowledge:

- Automaticity of multiplication facts. [Note: Division facts are not needed.]
- Proficiency in multiplying by multiples of 10 and 100 (e. g. 4 x 20),
- Understand division as splitting up into equal groups.
- Proficiency with 2-digit by 1-digit division (grade 4 outcome). Refer to Gr 4 Division algorithm for assistance if this is not in place.

SET SCENE stage

The problem task to present to students:

Have students play the '*How Many*' game. Students make two types of cards. One type is a 'how many groups are formed' kind of question. For example, the card could be: "how many boxes"; "how many bags"; "how many teams"; and so on. The other type of card is a 'how many in each group' kind of question. For example, the card could be: "how many zebras"; "how many peanuts"; "how many pencils"; and so on.

Organzie students into teams of about four students per team. Give each team one of the cards that were made. The task for each team is to:

1. Make up a division story that uses the question on the card as the basis for the problem in the story. For example, if the card is 'how many peanuts', the problem could be: "*Sam gave his 45 peanuts equally to 5 friends. How many peanuts did each friend get?*"
2. Write the number sentence for the problem.
3. Figure out the answer to the problem. Limit this to a specific time (e. g. 3 minutes).

Note:

Restrict the arithmetic complexity of problems to a maximum of 9 x 9 = 81; 81 ÷ 9 = 9.

When time is up, each team presents its solution. The other teams act as referees, indicating if something is wrong. If a team has detected an error, it has the opportunity to provide the correct solution.

There is another team challenge in the game. Certain phrases on the cards can be interpreted in two ways. For example, the phrase "how many people" can be seen as a 'how many in each group' kind of question if teams of people are being formed. The same question can be seen as a 'how many groups' kind of question if candies are being shared between people. When playing the game, either interpretation is allowed. Therefore, if a team provides a story about one type of question (e. g. how many groups are formed), another team can provide a story of the other type of question after the first team has finished.

Comments

The central purpose of the SET SCENE task is to focus on division problem solving. This provides a reason for learning more complex division arithmetic and a context for dealing with remainders.

DEVELOP stage

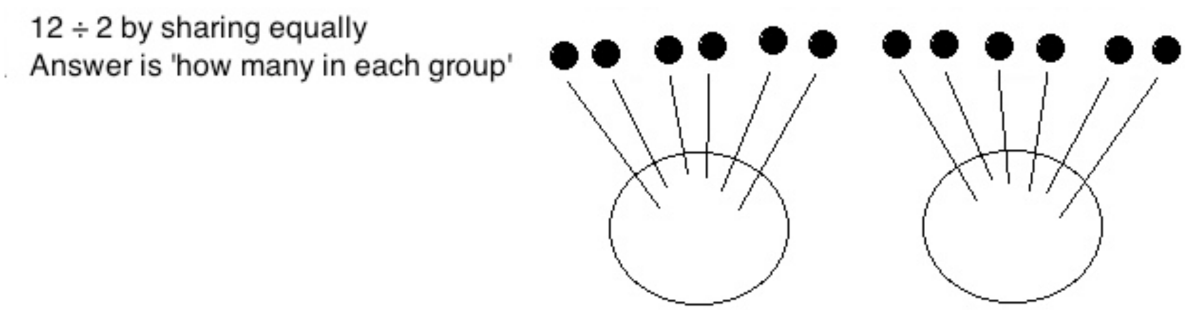
Comments

3-digit by 1-digit division can be done using the strategy of making equal groups (the answer is 'how many groups are made') OR the strategy of sharing equally (the answer is 'how many in the group').

Sample Problem (sharing equally):

12 candies are shared equally between 2 people. How many candies does each person get?

We know the number of groups is 2. The question concerns 'how many in each group?' The following diagram illustrates how to obtain an answer using manipulatives. One candy at a time is placed in each circle. This could be sped up by placing more than one candy at a time into each circle.



Sample Problem (making equal groups):

12 candies are shared equally so that each person gets 2 candies. How many people get the candies?

We know the size of the group is 2. The question concerns 'how many in each group?' The following diagram illustrates how to obtain an answer using manipulatives. It involves making groups of 2.

The strategy of sharing equally is a cumbersome way to think about division arithmetic. For example, to get an answer for 126 ÷ 9 by sharing equally, you would have to imagine nine empty circles and place one or more counters into each circle until all the counters have been placed into the circles.

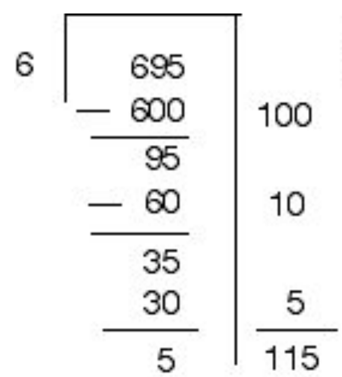
The strategy of making equal groups and keeping track of how many are made is a better way to think about division arithmetic. This strategy is used in the subtractive algorithm.

There are many writing styles for the subtractive. One writing style is shown here for the strategy of making equal groups. Students do not have to use that style. However, they do need to understand that the answer to a division question can be obtained by subtracting equal groups of the divisor and keeping track of how many groups are subtracted. Without this understanding, any personal strategy is as 'magical" as a calculator or as the "goes into" algorithm you learned when you were a student in elementary school.

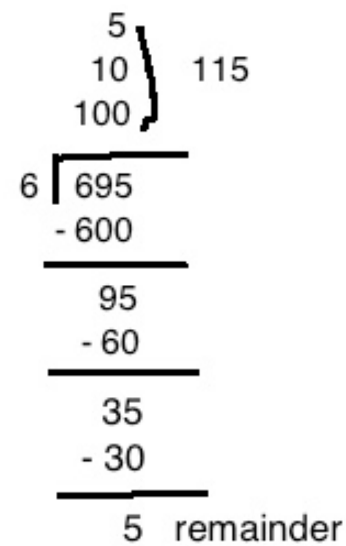
The subtractive algorithm is different from the ‘goes into’ algorithm. The subtractive algorithm has the advantages of: (1) not requiring division facts, (2) directly connected to the meaning of division as an action of splitting up into equal groups, and (3) having many ways to do it, and (4) making sense to most students.

See below for an example of the subtractive algorithm.

Two different writing styles for the algorithm are shown.



The answer to 695 ÷ 6 is 115 with remainder 5.



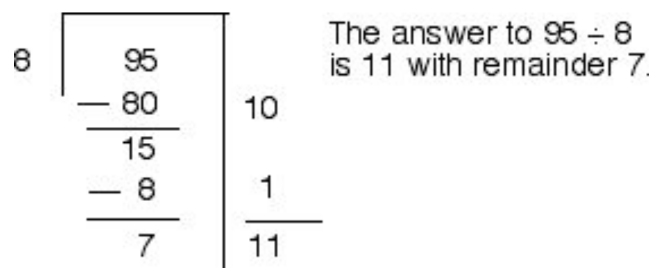
Notice that the division algorithm shown above involves subtraction, addition, and multiplication. Any efficient division algorithm, whether student-invented or otherwise, will involve these operations.

Activity 1: Revisits SET SCENE and addresses indicator 3.

- Revisit the SET SCENE task by asking students to present and discuss one of the division problems they made up (e. g. 45 ÷ 5 = ?).
- Change the numerical complexity of the division to a 3-digit divided by a 1-digit number (e. g. 145 ÷ 5 = ?). Ask students how they might obtain an answer to the division. Discuss their methods, including the calculator. Eventually tell students that they are going to learn how to do division with bigger numbers,

Activity 2: Addresses achievement indicators 2, and revisits grade 4 outcome.

- Provide students with about four 2-digit by 1-digit division tasks that sometimes involve remainders.
- Encourage them to obtain answers to the division in as fast a way as they can by thinking in terms of removing multiples of 10. Assist as necessary. [A fast way of obtaining the answer to 95 ÷ 8 is shown here. Note that a different writing style could be used instead.]



- Discuss what could be done with any remainders that occur.

Activity 3: Addresses achievement indicators 2 through 3.

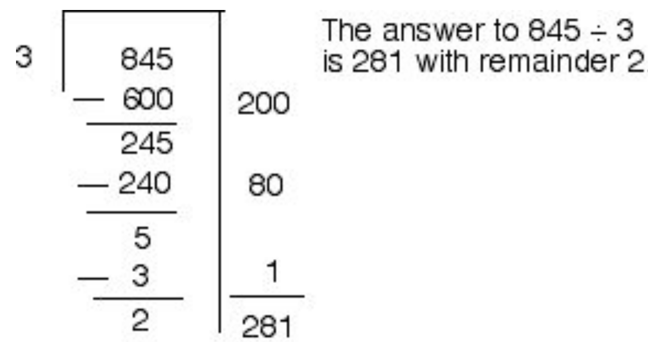
- Pose this problem: "*How many weeks in a year having 365 days?*" Students are likely to know the answer but that is okay (provides a check on the arithmetic method). Ask students to figure out a way of calculating the answer. Discuss their methods. Hopefully, someone will have tried to extend the grade 4 subtractive algorithm and writing style to calculate the answer. If not, you present and discuss an algorithm. Discuss making an appropriate use of the remainder. The correct answer is 52 weeks, not 52 weeks, remainder 1. [An example of ignoring the remainder.]
- Pose this problem: "*How many vans are needed to transport 200 students to an event if each van can carry 6 students?*" Ask students to calculate the answer. Discuss their methods, but emphasize the subtractive algorithm. Discuss making appropriate use of the remainder. The correct answer is 34 vans, not 33 remainder 2. [An example of rounding up the quotient (the division result).]

Activity 4: Addresses achievement indicators 2 and 3.

- Organize students into groups. Ask each group to create two 3-digit by 1-digit division problems (problem, not merely a division question) where one problem involves ignoring the remainder and the other problem involves rounding the quotient. Have each group solve its own problems.
- Have the groups exchange their problems with other groups. Have the groups solve the given problems. Have selected groups present their problems and solutions. Discuss results.

Activity 5: Addresses indicators 2, and practice.

- Provide students with three 3-digit by 1-digit division questions that sometimes involve remainders.
- Encourage students to obtain answers to the division in as fast a way as they can. Encourage them to think in terms of removing multiples of 10 and 100. Assist as necessary. [A fast way of obtaining the answer to 695 ÷ 6 is shown here.]



- Discuss what could be done with any remainders that occur.

Activity 6: Addresses indicators 2 and 3, and practice.

- Present a realistic problem involving a 3-digit number divided by a 1-digit number such as: "*Bob has 3 friends. He wants to give away his entire collection of 845 old CD's to his 3 friends. How many CD's will each friend get? Will Bob have any CD's left over?*"
- Discuss that the problem concerns the question 'how many in each group', rather than the question 'how many groups'. Discuss if it makes sense to use the subtractive algorithm to do the arithmetic. Ensure that students realize that thinking of 845 ÷ 3 as 'how many groups' or thinking of 845 ÷ 3 as 'how many in each group' would work out to be the same number. For this reason, we can use the subtractive algorithm to do the arithmetic '845 divided by 3'.
- Have students solve the problem, using the subtractive algorithm. Encourage them to obtain the answer to the division as fast as they can by thinking in terms of removing multiples of 10 and 100. Write fastest method on the board and discuss it.
- Present two other problems for which the answer involves 'how many in each group' and there is a remainder. Have students solve the problems, dealing with the remainders in a realistic way. Discuss their thinking.

Activity 7: Addresses indicators 2 and 3.

- Pose this problem: "*Suppose two people want to share candy bars equally. How many bars does each person get?*" Have students determine an answer. They should get 2 bars, with remainder 1. Ask them to write this remainder as a fraction. Hopefully, they will see it as 1/2. Discuss how the remainder '1' and the divisor '2)'are used to express the remainder as a fraction '1/2'. [The remainder 1 is divided by 2 to get 1/2 because the division question concerns dividing by 2.]
- Repeat about four times for other simple problems (e.g. *12 cookies shared equally among 5 people*). Each time students are expected to express the remainder in fraction form.

Activity 8: Addresses indicators 2 and 3.

- Pose this problem: "*At karate class Joe breaks a 23 cm board into two equal parts. How long is each part?*" Have students determine an answer. They should get 11 cm, with remainder 1. Ask them to write this remainder as a fraction. They should see it as 1/2. Ask students to write an equivalent fraction for 1/2 for which the denominator is 10 (expect 5/10). Ask them to write the 5/10 in decimal form. Discuss how a remainder can be expressed as a decimal. [Note: This should discourage thinking of any remainder of 3, for example, as being equivalent to .3.]
- Repeat about four times for simple problems involving a context of measurement or money where the denominator of the fraction is always a divisor of 10 (e.g. 234 cents shared equally among 5 people). Each time students are expected to express the remainder in decimal form.

Activity 9: Revisits SET SCENE and addresses indicators 2 and 3, and practice.

- Ask each group from SET SCENE to select one of the problems they made and to rewrite it so that it involves 3-digit by 1-digit division.
- Have each group solve their own problem.
- Have the groups exchange problems, with each group solving the problem it was given. Discuss results.

Activity 10: Assessment of teaching.

- Provide students with a 3-digit by 1-digit division problem that involves a realistic use of a remainder (e.g. *Joe has 127 CDs. He wants to store them in boxes having 8 CDs in each box. How many boxes will he need?*). Ask students to solve the problem, treating the remainder in a realistic way.
- Provide students with a 3-digit by 1-digit division problem that involves a remainder and a divisor that is a factor of 10. (e.g. *5 thirsty people want to share 232 litres of water equally*). Ask students to solve the problem, expressing the remainder as a decimal.

If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).

An example of a partial well-designed worksheet follows.

The worksheet contains a sampling of question types. More questions of each type are needed.

The MAINTAIN stage follows the sample worksheet.

Question 1.

Solve each problem. Treat the remainder in a real way.

a) Binki gives away 75 sports cards equally with 6 friends. How many cards does each friend get?

b) Binki is on a soccer team with a roster of 20 players. At practice, the coach ask the players to form teams of 3/ How many teams are possible?

c) Binki has a 635 toys to give away. He places them equally in ten boxes. How many toys are in each box?

d) Binki wants to put his collection of 504 CDs into boxes with 8 CDS in each box. How many boxes are needed?

Question 2.

Do the division, Write the remainder in fraction form.

a) $234 \div 6$

b) $338 \div 9$

c) $806 \div 5$

Question 3.

Do the division, Write the remainder in decimal form.

a) $293 \div 5$

b) $337 \div 2$

c) $806 \div 4$

MAINTAIN stage

Mini-task example

Every so often:

- Provide a 3-digit by 1-digit division problem (a problem, not merely a division question). Ask students to solve the problem. Discuss how the remainder should be treated.

Rich-task example

Using painting/drawing/processing software, pairs of students create two word problems (and the solutions) that involve 3-digit by 1-digit whole number division and that involve at least one of addition/subtraction/multiplication of whole numbers where the arithmetic complexity matches grade 5 expectations (e. g. 2-digit by 2-digit whole number multiplication). Students compile the problems into an illustrated word problem book and a corresponding solution book. Below is an example of a possible student-created problem. It involves multiplication, addition, and division to obtain the solution.

Sam and Martha loved candies. Sam had 12 cans with 48 candies per can in his closet. Martha, his sister, had 260 candies in a drawer. They decided to combine their candies and then store them in envelopes, with 8 candies placed in each envelope. How many envelopes will they need? Will there be any candies left over?

Comments

This is a rich-task because it integrates other mathematics, and writing and drawing.